Formal specification of opinions applied to the consensus problem

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Abstract. Agents are often lead to make collective decision (reaching a consensus about task or resource allocation, election, etc). This paper proposes a distributed protocol based on iterative exchange of opinions among strongly autonomous, weakly rational and heterogeneous agents. We prove that the protocol converges to a consensus while respecting agents' autonomy and fairness. First, a formalism to model agents' preferences, positions and opinions is developed. Then several operators required by the protocol are characterized (e.g. opinion cycle detector, aggregation operator, and consensus detector). Finally, the main experimental results we obtained when the protocol has been applied to the coalition formation problem in the e-commerce context.

keywords: intelligent agents, preference, consensus, coalition formation
topics: Multi-Agent Systems, Knowledge Representation, Decision Support Systems

1 Introduction

In Multi-Agent Systems (MAS), agents often have to reach a consensus. In e-commerce context, agents are strongly autonomous and weakly rational. Using this class of agents has several consequences on the design of protocols (as discussed in [1]). It is not possible to design a protocol that permits agents to act in any possible way; so we have to restrict their behavior. One way is to make assumptions about their internal state (e.g. some kinds of rationality, an utility or satisfaction function), but that is incompatible with autonomy. An other way is to give a protocol to agents that agreed on it, and to control that they abide by it: agents are autonomous because no hypothesis about their internal state is made.

In a consensus problem, agents' autonomy is conflicting with a solution computed using an external operator basically because: 1) known operators cannot generally lead to a solution that satisfies every agent (a good solution on average as in [2] is not agreed by all agents, since some of them could prefer to try to earn more, even if they risk to lose more); 2) generally (e.g. [3]) operators are based on internal informations about agents (utility function, inmost preferences), which violates their autonomy and privacy.

To reach a consensus among agents who are motivated to satisfy their own preferences first, we propose a protocol which may be summarized as follows: 1) agents may start with different preferences; 2) agents exchange data to influence others' opinions; 3) an iterative process that incites agents to evolve their opinion at run-time and guarantees

the consensus will be reached; 4) stops when a consensus is reached.
This protocol rises three questions. First: what is to be exchanged ? Using resources suppose that all services are convertible into them. Argumentation is often used to convince the other agents by giving new information as a rational way to reach a consensus; but such a process assumes that agents have symbolic computation capabilities [4], time and inference abilities [5, 6]. Such capabilities are not available for heterogeneous agents, and their normalization -as FIPA- is not practicable [1]. We choose to exchange agents' opinions represented as application, because: 1)it can be understood and processed by heterogeneous agents; 2)agents don't need to reveal their internal informations (which respects their autonomy). Over time, opinions should represent: 1)private preferences of an agent; 2)his current position (voluntarily influenced by other opinions) even if different from his private preferences; 3)a mean to influence other agents.
The second question is how to incite agents to modify their positions. Time cost could be used (by decreasing the incomes), but that implies that money is the only criteria used by agents to estimate a possibility. Here, agents are allowed to change their preferences until they give the same opinion twice (a cycle is detected, see section 4). To avoid infinite processing, agents have then the possibility to form alliances. An alliance is a group of agents that decide to behave as a super-agent; its opinion is computed using an aggregation operator on opinions of its members (see section 5). If nobody decides to form an alliance, the MAS chooses the two nearest agents w.r.t. their preferences (using an operator not presented in this paper due to the lack of place - annexe B) and force them to ally. Autonomy is not violated, because: i) this sanction is known and agreed by agents at the beginning of the protocol; ii) constraints on agents' behaviors are weak and can be checked.
The third question concerns the consensus legitimity. We could use a vote to decide about a consensus, but that could make the process tedious and slow down the convergence (since unsatisfied agents may vote against the reached consensus). So, we suggest to use a criteria that is known and agreed initially by agents.
In this paper, we propose an approach based on the exchange of opinions and their evolution among strongly autonomous and weakly rational agents (section 2). The protocol we propose requires: i) a formalism to represent and handle agents' opinions (section 3); ii) a cycle detector to recognize a cycle (section 4); iii) an aggregation operator that computes the opinion of an alliance or more generally a group of agents (section 5); iv) a chooser operator that computes the preferred possibility once a consensus is reached (section 6); v) a consensus detector able to decide that a consensus has been reached (section 7). Section 8 presents experiments and results. Related work are presented in section 9. Section 10 concludes our paper and outlines our future work.

2 Our approach

Conceptually speaking, in our protocol, two roles are distinguished (even if the same agent may play the two roles): the member who competes to perform tasks, and the representative of an alliance who plays the role of interface between his alliance and the other alliances, i.e. he receives opinions from his alliance's members, computes the

aggregated opinion and send it to the others.
The role of an Alliance's Member (AM) (Hypothesis: to begin with, each agent creates an alliance with cardinality 1; in this first case, the agent supports the two roles: member and representative.)
Main:

◇ position $\omega = private\_preference(AM)$
◇ AM sends his position ω and in the same time receives the positions of other agents
◇ while a consensus is not reached do /*$\neg \bowtie$*/
  ◇ if a cycle is detected then /*Θ = True*/
    ◇ then AM calls alliance formation
  ◇ AM computes his new position ω
  ◇ AM sends his position ω to his representative
◇ endwhile

Alliance formation:

◇ process of proposition/acceptation of alliance formation
◇ if no alliance is formed
  ◇ then the two nearest alliances ally /*chosen using nearest alliances chooser. This operator is based on a kind of distance between aggregated opinions of alliances. It chooses the two alliances having the minimal distance. Unfortunetly, this operator is not presented in this paper because of the lack of space.*/

The role of an Alliance's Representative (AR)
Main:

◇ AR receives the positions from the members of his alliance
◇ AR computes the alliance's position /*using aggregation operator $\amalg$*/
◇ AR broadcasts the position of the alliance

3 Opinions

Notation: let A be the set of agents. Lower-case letters $a, b, \ldots$ denote agents. S is the set of possibilities, Δ the set of degrees of preference, ς the set of level of conflicts and Ω the set of opinions. We will call $View$ the set of views and H the set of histories (a turn number t is in $[1, T]$).
The preference for a possibility a over a possibility b is the expression of the intensity of its author's will to have a chosen instead of b. Then an on opinion is a set of preferences comparing every possibilities to every other one. To represent preferences, we propose to use degrees that range from -1 to 1: the closer to 1 a degree is, the more the first possibility is preferred to the second (and reciprocally). We don't use an order even partial as in the case of the most other approaches (see section 9), basically because the transitivity is not an inherent property of preferences in rational context. For example, an agent has the choice between three cars with two criteria, the price and the consumption ($c_1 = (\$10K, 8L)$, $c_2 = (\$14K, 6L)$ and $c_3 = (\$18K, 4L)$) and has the following rational rule: "If consumptions are close (less than $3L$), I choose the most expensive car (because it is more luxuous); else I choose the one that consumme the less."; the results are: $c_1 \preceq c_2$, $c_2 \preceq c_3$, and $c_3 \preceq c_1$, what is not transitive.
What happens at the group level ? The first idea is to compute the mean of the degrees

($\omega_{i,j}^{\{a,b\}} = (\omega_{i,j}^{a} + \omega_{i,j}^{b})/2$), but this formula leads to strange results: a preference of two agents with opposite degrees equals zero (i.e. indifference), while incompatible preference could be find. In fact, computing the average for a group leads to the the loss of too much information. To solve this problem, we propose to use the standard deviation that summarizes the dispersion of values.
In order to modelize finely these concepts about possibilities, we propose to distinguish different levels. Our formalism of opinions should be used to represent the agent's private opinions, his computed positions and the exchanged positions of agents and alliances.

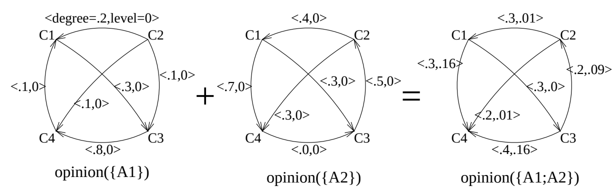

Fig. 1. Opinions and aggregation

Our formalism. A preference between two possibilities i and j is defined by a degree of preference $\delta_{i,j}$ and a level of conflict $\sigma_{i,j}$ (standard deviation).

Definition 1 (Opinion sets).
The set of possibilities is S, the set of degrees is $\Delta = [-1, 1]$ and the set of levels of conflict is $\varsigma = [0, 1]$.

Property 1 (Degree set). The set of degrees is: i) stable when computing the opposite; ii) continuous; iii) contains a unique element 0 that represents the indifference.

Interpretation:
– A degree $\delta_{i,j}$ between i and j is interpreted as follows:
• $0 < \delta_{i,j} \leq 1 \iff$ "I prefer i to j with a degree $\delta_{i,j}$"
• $-1 \leq \delta_{i,j} < 0 \iff$ "I prefer j to i with a degree $-\delta_{i,j}$"
• $\delta_{i,j} = 0 \iff$ "I have no preference between i and j"
– A level of conflict $\sigma_{i,j}$ between i and j is interpreted as follows:
• $\sigma_{i,j} = 0 \iff$ "everybody agrees the degree of preference" (low level of conflict)
• $\sigma_{i,j} = 1 \iff$ "the maximum level of conflict is reached"
• $\sigma_{i,j} < \sigma'_{i,j} \iff$ "opinion with level of conflict $\sigma_{i,j}$ is less conflicting than opinion with level $\sigma'_{i,j}$"

Definition 2 (Opinion). An opinion $\omega \in \Omega$ is an application $\omega : S \times S \rightarrow \Delta \times \varsigma$ with the following properties:
– $\forall i \in S$, $\omega_{i,i}^{a} = \langle 0, 0 \rangle$: a is indifferent to i and i;
– $\forall (i, j) \in S^2$, $\omega_{i,j}^{a} = \langle \delta, \sigma \rangle \Rightarrow \omega_{j,i}^{a} = \langle -\delta, \sigma \rangle$: the degree of preference is antisymmetric.

4 Cycle detector

In order to be sure that the process finishes, we have to detect when a situation happens twice (i.e. a cycle).
Characterization. The idea is to save the history (process) of the exchanged opinions and to detect similar situations called "views" (notation: $u \approx_v v$) thanks to the operator called "cycle detector" as follows.

Definitions 3 View: A view v is an application $A \to \Omega$.
History: An history $h \in H$ is a sequence $(v_t)_{1 \leq t \leq T}$ of views, where T is the length of the history.
Partial order on opinions: A partial order on opinions $\succ_o$ is defined by: $\forall(\omega^a, \omega^b) \in \Omega^2, \omega^a \succ_o \omega^b \iff \forall(i,j) \in S^2, \delta^a_{i,j} \geq \delta^b_{i,j} \wedge \sigma^a_{i,j} \leq \sigma^b_{i,j}$.
Partial order on views: Let $(\omega^a)_{a \in A}$ the agents' opinions. A partial order on views $\succ_v$ is defined by: $\forall(v, v') \in V^2$, $v \succ_v v' \iff \forall(a,b) \in A^2, \omega^a \succ_o \omega^b$.

Definition 4 (Cycle detector). A cycle detector Θ is an application $H \times \mathbb{R}^* \times \mathbb{R}^* \to \{\text{False, True}\}$ characterized as:
i) $\forall h \in H, h = (v_t)_{1 \leq t \leq T}, \exists t \in [1, T[, \forall a \in A, v_t(a) = v_T(a) \Rightarrow \Theta(h) = \text{True}$ detects true cycles (i.e. when a situation happens twice);
ii) $\forall(u,v) \in V^2$, $u \approx_v v \Rightarrow \forall(u', v') \in V^2, u \succ_v u' \succ_v v' \succ_v v, u' \approx_v v'$: if u and v correspond to a cycle, then all the couples of views (u', v') situated between u and v must be detected as cycles too.

Example of our distance cycle detector.

Definitions 5 Opinion preference distance: An opinion preference distance $|.,.|^p_o$ is an application $\Omega \times \Omega \to \mathbb{R}$ defined by: $\forall(\omega, \omega') \in \Omega^2$, $|\omega, \omega'|^p_o = \max_{i,j} |\delta_{i,j} - \delta'_{i,j}|$.
Opinion conflict distance: An opinion conflict distance $|.,.|^c_o$ is an application $\Omega \times \Omega \to \mathbb{R}$ defined by: $\forall(\omega, \omega') \in \Omega^2$, $|\omega, \omega'|^c_o = \max_{i,j} |\sigma_{i,j} - \sigma'_{i,j}|$.
View preference distance: A view preference distance $|.,.|^p_v$ is an application $V \times V \to \mathbb{R}$ defined by: $\forall(v, v') \in V^2, |v, v'|^p_v = \max_{a \in A} |\omega^a_v, \omega^a_{v'}|^p_o$.
View conflict distance: A view conflict distance $|.,.|^c_v$ is an application $V \times V \to \mathbb{R}$ defined by: $\forall(v, v') \in V^2, |v, v'|^c_v = \max_{a \in A} |\omega^a_v, \omega^a_{v'}|^c_o$.

Definition 6 (Distance cycle detector).
Let $(\epsilon_p, \epsilon_c) \in \mathbb{R}^{*2}$ be two thresholds.
$\breve{\Theta}$ is an application $H \times \mathbb{R}^* \times \mathbb{R}^* \to \{\text{False, True}\}$ defined by:
$\forall h \in H$, $h = (v_t)_{1 \leq t \leq T}$, $\breve{\Theta}(h)$=True$\iff \exists t \in [1, T-1]$,
$|v_t, v_T|^p_v \leq \epsilon_p \wedge |v_t, v_T|^c_v \leq \epsilon_c$.

$\breve{\Theta}$ returns true if the two views are close enough considering both view preference distance (ϵ_p) and view conflict distance (ϵ_c).

Proposition 1. $\breve{\Theta}$ is a cycle detector.

Proof. i) $v_t = v_T \Rightarrow |v_t(a), v_T(a)| = 0 \Rightarrow \breve{\Theta} = $ Trueii) we can prove that $u \approx_v v \wedge u \succ_v u' \succ_v v' \succ_v v \Rightarrow |u', v'|_v^p \leq |u, v|_v^p \wedge |u', v'|_v^c \leq |u, v|_v^c \Rightarrow u' \approx_v v'$.

5 Aggregation operator

The main interest of our opinion model is its ability to compute naturally the opinions of a group contrary to other approaches. In fact, using a total order to modelize individual preferences prevents from computing groups' preferences with the same formalism. For example, if a_1's preference is $1 > 2$, and a_2's preference is $2 > 1$, what is the preference of a_1, a_2 ?
In our framework and in MAS in general, opinions' aggregation is usefull to: i) estimate the opinion of a group, what may be used to choose which actions to be performed; ii) compute the new position of an agent (others' opinions are informations that an agent should take into account in order to evolve his private opinion). A way to do that is to aggregate the opinions of others with small weights (using a weighted aggregation, as defined in section 9).
Characterization. According to the rationality of the aggregation, we propose axioms necessarily respected by the aggregation operator.

Definition 7 (Aggregation operator). Let $n \in \mathbb{N}^*$.
An aggregation operator $\amalg_n$ is an application $\Omega^p \rightarrow \Omega$ with the following properties:
i) [Independence] $\amalg_n(\omega^1_{i,j},\ldots,\omega^n_{i,j}) = f(\omega^1_{i,j},\ldots,\omega^n_{i,j})$: the aggregation of two opinions on two possibilities doesn't depend on opinions on other possibilities;
ii) [Everywhere defined] $\forall(\omega^1,\ldots,\omega^n) \in \Omega^n$, $\forall(i,j) \in S$, $\amalg_n(\omega^1_{i,j},\ldots,\omega^n_{i,j})$ is defined: all opinions could be aggregated;
iii) [Keep equality] $\amalg_2(\langle\delta,\sigma\rangle,\langle\delta,\sigma'\rangle) = \langle\delta,\sigma''\rangle$: the aggregation of two same degrees equals the degrees;
iv) [Equity] $\forall\tau$ a permutation on $[1,n]$, $\amalg_n(\langle\delta_1,\sigma_1\rangle,\ldots,\langle\delta_n,\sigma_n\rangle) = \amalg(\langle\delta_{\tau(1)},\sigma_{\tau(1)}\rangle,\ldots,\langle\delta_{\tau(n)},\sigma_{\tau(n)}\rangle)$: the result of the aggregation is equitable, it doesn't depend on the order of opinions;
v) [Opposition] $\amalg_2(\langle\delta,\sigma\rangle,\langle-\delta,\sigma'\rangle) = \langle 0,\sigma''\rangle$: if two agents have opposite possibilities, then the result of aggregation is that the degree of preference is null (but not the level of conflict);
vi) [Associativity] $\amalg_2(\amalg_2(\omega,\omega'),\omega'') = \amalg_2(\omega,\amalg_2(\omega',\omega''))$: the opinion of an aggregated opinion must not depend on how the group has been formed (e.g. when agents join the group)

Example of our aggregation operator.

Definition 8 (Aggregation of groups' opinions). Let $(\omega_i)_{1\leq i\leq n}$ be a sequence of opinions: $\forall i, \omega_i = \langle\delta_i,\sigma_i\rangle$.
The quadratic mean is defined by: $\forall i, \overline{m}_i = \sigma_i^2 - \delta_i^2$.
We define $\breve{\amalg}((\omega_i)_{1\leq i\leq n}) = \langle\delta,\sigma\rangle$ where: $\delta = \frac{1}{n}\sum_{i=1}^n \delta_i$, $\overline{m} = \frac{1}{n}\sum_{i=1}^n \overline{m}_i$ and $\sigma = \sqrt{\overline{m} - \delta^2}$

Remark 1. Formulae are not randomly chosen, but are consequences of formulas used in statistic. Given a standard deviation σ, m a mean and $\overline{m}$ a quadratic mean, from the Huygens/König formula, we deduce: $\sigma = \sqrt{\overline{m} - m^2}$. In this paper, $m = \delta$, so $\sigma = \sqrt{\overline{m} - \delta^2}$. The same formula are used to compute $\overline{m}_i = \sigma_i^2 - \delta_i^2$.

Proposition 2. $\breve{\Pi}$ is an aggregation operator.

Proof. Computing the equalities above (def.7) is enough to prove this. (see Annex A).

An example of aggregation is given in figure 1. The opinions of the two agents at the left are aggregated into one opinion (the right one). Let us remark that the levels of conflict that vary from 0 to 0.16, depend on the closeness of degrees of preferences.

Definition 9 (Weighted aggregation). Let $p \in \mathbb{N}^*$.
A weighted aggregation operator $\tilde{\Pi}$ is an application $(\Omega \times \mathbb{R}^+)^p \to \Omega$ defined by: $\tilde{\Pi}((\omega_1, w_1),\ldots,(\omega_p, w_p))$ aggregates all opinions, replacing the degrees δ_i by $w_i \times \delta_i$ and the level of conflict σ_i by $w_i \times \sigma_i$.

6 Chooser operator

When a consensus is reached (the agents have close opinions), we have to find the preferred solution. This is why opinions will be aggregated using the chooser operator defined below. When preferences are formalized by a total order, there is a unique possibility preferred to all others. In partial orders, several maximal possibilities may exist. As we allow cycles (in the preference relation), generally there is no maximal preferred possibility. The difficulty is that we have to find a compromise between maximizing the degrees of preference and minimizing the level of conflict (w.r.t. other possibilities).
Characterization. The necessary axiom of a chooser operator is that if a best possibility exists, then this possibility will be chosen.

Definition 10 (Chooser operator). Let $E_{max} = \{i \in S/[\forall j \in S, \delta_{i,j} \geq 0] \wedge [\forall (k,l) \in S^2, (\delta_{i,j} \geq \delta_{k,l}) \wedge (\sigma_{i,j} \leq \sigma_{k,l})\}$.
A chooser operator $\bigcirc$ is an application $\omega \to S$ with the property: if $E_{max} \neq \emptyset$, then $\bigcirc(\omega) \in E_{max}$

Remark 2. Generally, E_{max} is empty; so we defined several heuristics to make this choice. In the following, we present one of them called "degrees first, conflicts next"

Example of our chooser.

Definitions 11 Weight of a possibility: We call weight of a possibility $i \in S$ for the opinion ω the value $w_\omega(i) = \frac{1}{|S|-1} \sum_{j \in S \setminus \{i\}} \delta_{i,j}$.
Efficient opinion: An opinion $\langle \delta, \sigma \rangle$ is efficient if $\nexists \langle \delta', \sigma' \rangle, \delta \geq \delta' \wedge \sigma \leq \sigma' \wedge (\delta > \delta' \vee \sigma < \sigma')$.

It's difficult to take into account the degree of preference and the conflict level in the same time, because we don't know which criteria must be used before the other; in this heuristics, we favor degrees.

Definition 12 (Degrees first, conflicts next). – Step 1: Build the set of the best possibilities (I) as follows:
Let $(w_i)_{i \in S}$ be the sequence of weights of possibilities of S computed using def.11.
Let $w_{max} = \max_i w_i$ and let $\epsilon \in \mathbb{R}^*$ be a threshold.
Let $I = \{i \in [1, n] / w_i \geq w_{max} - \epsilon\}$ be the set of possibilities that are close to the maximum.
– Step 2: K is a restriction of I such that K is a total order
Let $\succeq_{\mathcal{P}}$ be the preference relation defined by $\delta_{i,j} \geq 0 \Longleftrightarrow i \succeq_{\mathcal{P}} j$.
Let Q be the set of relations between possibilities of I ordered by σ_i.
Let apply the process:
1- Let K be an empty partial order.
2- while $Q \neq \emptyset$ do
3- let $(i \succeq_{\mathcal{P}} j) = min(Q)$; $Q \leftarrow Q \backslash \{i \succeq_{\mathcal{P}} j\}$. /* less conflict*/
4- If $K \cup (i \succeq_{\mathcal{P}} j)$ doesn't contain a cycle, then add the relation to K.
5- endwhile
We call "degrees first, conflicts next" the application $\omega_S \mapsto max(K)$.

Proposition 3. The application "degrees first, conflicts next" is an opinion chooser.

Proof. In order to compute $max(K)$, we have to prove that K is a total order. Then we have to prove that if $Emax$ is not empty, then an element $i^\star$ of $Emax$ will be chosen. The leading idea behind the proof: i) $i^\star \in I$; ii) the relations that contain $i^\star$ are added before the others; iii) $max(K) \in E_{max}$. (see annex C).

Example: let us apply this operator to the aggregated opinion of the figure 1. Step 1: let us compute the sequence of weights: $w_1 = (-.3 + .3 + .3)/3 = .1$, $w_2 = (.3 - .2 + .2)/3 = .1$, $w_3 = (-.3 + .2 + .4)/3 = .1$ and $w_4 = (-.3 - .2 - .4)/3 = -.3$; so $I = \{C_1, C_2, C_3\}$. Step 2 : $Q = \{C_1 \succeq_{\mathcal{P}} C_3; C_2 \succeq_{\mathcal{P}} C_1; C_3 \succeq_{\mathcal{P}} C_2\}$ (remark: the most important relations form a cycle); $K_0 = \emptyset$, $K_1 = \{C_1 \succeq_{\mathcal{P}} C_3\}$, $K_2 = \{C_1 \succeq_{\mathcal{P}} C_3; C_2 \succeq_{\mathcal{P}} C_1\}$ and $K_3 = \{C_1 \succeq_{\mathcal{P}} C_3; C_2 \succeq_{\mathcal{P}} C_1\}$. Finally, C_2 is the preferred car.

7 Consensus detector

A consensus operator has to answer the question: do all agents agree ? The vote is often used: firstly, each agent chooses one possibility and the one that has the maximum of votes is elected. In some vote systems, agents may choose more than one possibility (often two), but all possibilities have the same weight. We propose to extend this system by aggregating all opinions into a lone one using an aggregation operator, and then by choosing a possibility by applying a chooser operator on the aggregated opinion.
Two parameters are taken into account: the degree of preference and the conflict level.
Characterization.

Definition 13 (Consensus detector). A consensus detector $\bowtie$ is an application $\Omega \rightarrow$ {False, True} defined by: $\bowtie(\omega) = \text{True} \Longleftrightarrow \forall (i, j) \in S^2, \sigma_{i,j} = 0$

It seems rational to impose that if one possibility is preferred by all agents, then this possibility will be elected.
Example of consensus detector.

Definition 14 (Epsilon consensus detector). Let $\epsilon \in \mathbb{R}^*$.
An epsilon consensus detector $\breve{\triangleleft}_\epsilon$ is an application $\Omega \rightarrow$ {False, True} defined by:
$\forall (i,j) \in S^2, \sigma_{i,j} \leq \epsilon \Rightarrow \bowtie (\omega) = \text{True}$

Proposition 4. For all $\epsilon \in \mathbb{R}^*$, $\breve{\triangleleft}_\epsilon$ is a consensus detector.

Proof. The proof is obvious: $\sigma_{i,j} = 0 \Rightarrow \sigma_{i,j} \leq \epsilon \Rightarrow \bowtie (\omega) = \text{True}$

8 Experiments

We distinguish the coalition from alliance: some agents may ally because they are interested in the same solution, even if they don't collaborate in a coalition to fulfill a task.
Protocol of experiment. We have made several experiments, but we present here only the most significant result.
In a reaching consensus problem, it is difficult to find an efficient strategy. In order to test our formalism and operators, we have built a family of strategies and organized a tournament between these strategies.
The problem chosen to test our strategies is an allocation of tasks in an e-commerce context (see [1, 7]). Some sub-tasks have to be allocated to several agents who are not able to fulfill all tasks because they have limited skills (no agent meets all the requirements of a task). In our study, 7 agents have to share 8 sub-tasks among themselves, and 32 possibilities are assumed available.
Each agent chooses to take the others' opinions into account with a more or less great weight. At the beginning, it is in their interest to be rigid (i.e. do not take others' opinions into account) in order to influence the opinions of the others. At the end, they should better be flexible in order to have chance to be assigned a task. The question is: at what speed do I decrease my rigidity ? We define a strategy as a speed of decreasing.
Formally, the rigidity r is defined by: $\forall a \in A$, $\forall \alpha \in [0,1]$, $\forall t \in [1,T]$,$r(t) = \exp^{-\alpha t}$.
The agent computes his new opinion as follows:

- first, he aggregates the opinions of other agents: $\omega_m = \amalg(\{\omega'_b / b \in A \backslash \{a\}\})$;
- then he applies the weighted aggregation operator to aggregate his preferences weigthed by r and other agents' preferences weigthed by $1-r$; as result, the strategy is defined by:

$s^a_\alpha(t/10) = \tilde{\amalg}(< \omega_a, r >, < \omega_m, 1-r >)$.
Each strategy α (used by one agent) is opposed to a strategy β (used by all other agents, i.e. uniform population). For each fight α against β, we compute the ratio of income (in comparison with the agent's maximal income) for the agent using the strategy α.
The results are presented as follows (see figure 2): the strategy α takes place on the X-axis, and the mean of percentages of income (for all agents that used the strategy α) on Y-axis. Each curve represents the set of results for a fixed value of β ($\beta \in [0.0, 0.2, \ldots, 1.0]$ represents strategies of other agents).
Results. Figure 2 gives the results of our experiments. It shows that, whatever the

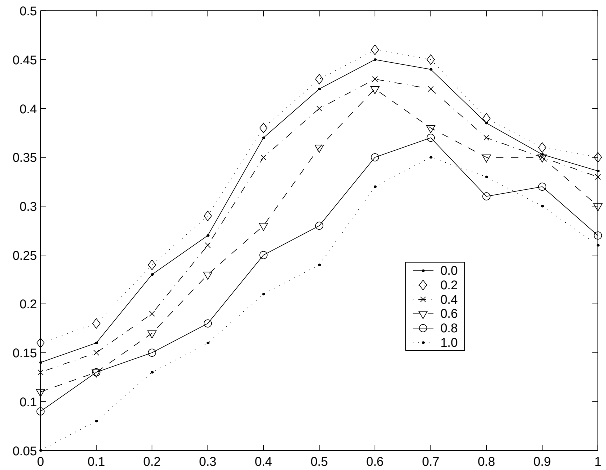

Fig. 2. Results of experiment

value of β, the best strategy α remains around $\alpha = 0.7$. This result is interesting as it emphasizes that our protocol doesn't favor extremely rigid strategies. Such strategies don't lead to rational solutions, as when every agent is extremely rigid, the final solution is given by the chooser applied to the aggregated opinions at the first step, what amounts to use an external chooser operator. Symetrically, too flexible strategies, as one can expect, are not interested.

9 Related work

In [8], Kenneth Arrow tackles the problem of finding a function of aggregation of preferences that respects the intuitive idea of such aggregation. The concept of preference is formalized as a binary relation $<$ that is antisymmetric, transitive and not reflexive. Based on this modelization, he proves that the only solution is a totalitarian system (one agent decides). This very strict modelization is not rich enough to represent some aspects of preferences: i) the indifference (no preference) is not modelized; ii) there is no level of preference (no intermediate degree); iii) a rational preference relation may be non-transitive (see section 3).
Many representations of preferences have been proposed in order to solve the impossibility problem of K. Arrow [9]: i)as a preference ordering of the possibilities what results to a total order; ii)as a utility function: the result is a total order, but with a measure of the difference of preference between two possibilities that is richer than preferred/not preferred; iii)as a preferred relation with some degree of preference of any alternative over another: the degree is interpreted as a degree of credibility.
The modelization of users' preferences [10] is based on several kinds of transitivities (more or less strict: min-transitivity, weak-transitivity, etc.) and two symbols that represent indifference and incomparability. We consider that the transitivity is necessary for a total order, but not for preference's modelization: in fact, K. Arrow's modelization

refers to an absolute judgment, while a preference is relative.
We prefer to talk about "opinion", because preferences refer to internal choice, while in this paper opinion is used as private preference, expressed position and a way to influence others (section 1).
Our formalism may be viewed as a generalization of several others. If we limit values of degrees to $\{-1, 0, 1\}$ and don't take the level of conflict into account, our formalism is equivalent to a total order (strict if we remove the 0). Incomparability when added leads to a partial order; in our approach, the semantics of the incomparability is a high level of conflict. To represent a valuation, we have to impose the constraint: $\delta_{i,j} \geq 0 \wedge \delta_{j,k} \geq 0 \Rightarrow \delta_{i,k} = \delta_{i,j} + \delta_{j,k}$.
Some approaches allow agents to use infinite value for degrees of preference, it allows agents to represent the refute or to impose a choice, as a veto. A veto reduces the set of possibilities, what may be decided before a consensus process.

10 Conclusion and future works

This paper introduces a new formalism of opinions and shows how use it in a consensus protocol for tasks' allocation among strongly autonomous, weakly rational and heterogeneous agents. This formalism permits fine representations of an agent's opinions (several degrees of preference and uncertainty) and of a group's one (several degrees of preference and levels of conflict) thanks to the aggregation operator.
In the future, we will test other operators and more complex strategies in order to show the richness of our formalism.

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A Proof of that $\breve{\amalg}$ is an aggregation operator

Proof. $\breve{\amalg}$ is an aggregation operator i) Independence: obvious;
ii) Everywhere defined: obvious;
iii) Keeps equality: since $\delta = \frac{1}{n}\sum_{i=1}^{n} \times\delta_i$, $\exists\delta'\forall i\delta_i = \delta' \Rightarrow \delta = \delta'$;
iv) Equity: since δ and $\overline{m}$ only depend on sums of δ_i and $\overline{m}_i$, and since the operator $\sum$ is commutative, then the result doesn't depend on the order of opinions;
v) Opposition: $\breve{\amalg}(\langle\delta,\sigma\rangle,\langle-\delta,\sigma'\rangle) = \langle 0, \sqrt{\frac{1}{2}(\sigma^2 + \sigma'^2 - 2\delta^2)}\rangle$
vi) Associativity: let $\langle M_1, \sqrt{\overline{M}_1 - M_1^2}\rangle = \amalg(\omega,\omega')$, where $M_1 = (\delta + \delta')/2$, and $\overline{M}_1 = (\overline{m} + \overline{m}')/2$;
let $\langle M_2, \sqrt{\overline{M}_2 - M_2^2}\rangle = \amalg(\omega',\omega'')$, where $M_2 = (\delta' + \delta'')/2$, and $\overline{M}_2 = (\overline{m}' + \overline{m}'')/2$.
Then let $\langle M_3, \sqrt{\overline{M}_3 - M_3^2}\rangle = \amalg(\amalg(\omega,\omega'),\omega'')$, where $M_3 = (M_1 + \delta'')/2$, and $\overline{M}_3 = (\overline{M}_1 + \overline{m}'')/2$;
and let $\langle M_4, \sqrt{\overline{M}_4 - M_4^2}\rangle = \amalg(\omega, \amalg(\omega',\omega''))$, where $M_4 = (\delta + M_2)/2$, and $\overline{M}_4 = (\overline{m} + \overline{M}_2)/2$.
Now, we have just to prove that $M_3 = M_4$ and $\overline{M}_3 = \overline{M}_4$:
$M_3 = (M_1 + \delta'')/2 = (\delta + \delta' + \delta'')/3 = (\delta + M_2)/2 = M_4$
$\overline{M}_3 = (\overline{M}_1 + \overline{m}'')/2 = (\overline{m} + \overline{m}' + \overline{m}'')/3 = (\overline{m} + \overline{M}_2)/2 = \overline{M}_4$

B Nearest opinion chooser

Characterization. We are not always able to determine the two nearest opinions, because it is difficult to define a distance on opinions. To characterize a nearest opinion chooser, we need to represent the fact that two opinions that are more near for all couple of possibilities (for their degrees of preference and for their level of conflict) than two others will be preferred (but not necessarily chosen). So we will first compute vectorial deviations between two opinions, and then partially order these couples to find the best ones.

Definitions 15 Efficient Vectors: Let $p \in \mathbb{N}^*$ and let E^p be a vectorial space. Let $\preceq$ a partial order on E^p defined by $\forall(u,v) \in E^p$, $u \preceq v \iff \forall i \in [1,p], u_i \leq v_i$. The efficient vectors of E^p are the maximal elements of $(E^p, \preceq)$.
Vectorial Total Deviation: Let S be a support and let ω and ω' be two opinions with support S. Let $n = |S|$. A Vectorial Total Deviation $\langle.,.\rangle$ is an application $\omega \times \omega' \to \Delta^{n(n-1)/2} \times \varsigma^{n(n-1)/2}$ defined by: $\langle\omega,\omega'\rangle = (|\delta_{1,2} - \delta'_{1,2}|, \ldots, |\delta_{1,n} - \delta'_{1,n}|, |\delta_{2,3} - \delta'_{2,3}|, \ldots, |\delta_{2,n} - \delta'_{2,n}|, \ldots, |\delta_{n-1,n} - \delta'_{n-1,n}|, |\sigma_{1,2} - \sigma'_{1,2}|, \ldots, |\sigma_{1,n} - \sigma'_{1,n}|, |\sigma_{2,3} - \sigma'_{2,3}|, \ldots, |\sigma_{2,n} - \sigma'_{2,n}|, \ldots, |\sigma_{n-1,n} - \sigma'_{n-1,n}|)$.

Definition 16 (Nearest opinions chooser). A nearest opinions chooser is an application $\mathcal{B} \to \Omega^2$ with the constrain: given the set of vectorial total deviation VTD computed on $\mathcal{B}\times\mathcal{B}$ ($VTD = \{\langle\omega,\omega\rangle/(\omega,\omega) \in \mathcal{B}^2\}$), given the set of efficient opinions of VTD namely VTD_{eff}, $\Psi(B) \in VTD_{eff}$: we must not be able to find two nearest opinions closer than the two chosen opinions.

Remark 3. $\exists(\omega, \omega') \in \Omega^2, \omega = \omega' \Rightarrow (\Psi = (\omega'', \omega''') \Rightarrow \omega'' = \omega''')$: if there exists two equal opinions, then the chosen opinions will be equal too.

Example of our norm nearest opinion chooser.

Definitions 17 Vectorial Preference Deviation: Let S a support and ω and ω' two opinions with support S. Let $n = |S|$. A vectorial preference deviation $\langle ., . \rangle_p$ is an application $\Omega \times \Omega \rightarrow \Delta^{n(n-1)/2} \times \varsigma^{n(n-1)/2}$ defined by: $\langle \omega, \omega' \rangle_p = (|\delta_{1,2} - \delta'_{1,2}|, \ldots, |\delta_{1,n} - \delta'_{1,n}|, |\delta_{2,3} - \delta'_{2,3}|, \ldots, |\delta_{2,n} - \delta'_{2,n}|, \ldots, |\delta_{n-1,n} - \delta'_{n-1,n}|)$.
Vectorial Conflict Deviation: Let S be a support and let ω and ω' be two opinions with support S. Let $n = |S|$. A vectorial conflict deviation $\langle ., . \rangle_c$ is an application $\Omega \times \Omega \rightarrow \Delta^{n(n-1)/2} \times \varsigma^{n(n-1)/2}$ defined by: $\langle \omega, \omega' \rangle_c = (|\sigma_{i,j} - \sigma'_{i,j}|)_{1 \leq i < j \leq n}$.
Square Norm: Let $p \in \mathbb{N}^*$ and let E^p be a vectorial space. The usual square norm $||.||$ is defined by: $\forall v \in E^p, ||v|| = \sqrt{\sum_{i=1}^{p} v_i^2}$

Definition 18 (Norm Nearest Opinions Chooser). Let $p \in \mathbb{N}^*$.
A square norm nearest opinions chooser $\ddot{\Psi}$ is an application $\mathcal{B} \rightarrow \Omega^2$ defined by:
i) let E_{min} be the set of couples of opinions (ω, ω') such that $||\langle \omega, \omega' \rangle_p||$ is minimal;
ii) if E_{min} contains more than one couple, then let remove couples of opinions (ω, ω') such that $||\langle \omega, \omega' \rangle_c||$ is minimal;
iii) if E_{min} contains more than one couple, then let choose a couple randomly.

Proposition 5. The application $\ddot{\Psi}$ is a nearest opinion chooser.

Proof. if two opinions are nearer, then it means that all coordinates are less or equal, what leads necessarily to a smaller norm.

C Proof that "degrees first, conflict next" is a chooser operator

Firstly, let us proof that K is K is a partial order, because it is an empty order in which we only add relations that don't add cycles. Why is it a total order ? For all $(i, j) \in S^2$, we try to add $i \succeq_{\mathcal{P}} j$ or $j \succeq_{\mathcal{P}} i$. If the relation is refused, it means that there exists a sequence of possibilities between i and j, ordered in the opposite order than the refused relation. So i and j are already comparable. Since we try to add all relations ($i \succeq_{\mathcal{P}} j$ or $j \succeq_{\mathcal{P}} i$), the order transitively closed is total.
Secondly, let us proof that $max(K) \in E_{max}$. Let $i^\star \in E_{max}$:
i) $w_i = \sum_j \delta_{i,j}$ and $\forall(i, j, k) \in S^3, \delta_{i^\star,j} \geq \delta_{k,l}$, so $w_i^\star$ is maximal, so $E_{max} \subset I$ (all elements of E_{max} have the same weight);
ii) $\forall(i, j, k) \in S^3, \sigma_{i^\star,j} \leq \sigma_{k,l}$, so the relations that contain an element $i^\star$ of E_{max} are added before the others;
iii) K contains all relations that contain at least one element $i^\star$ of E_{max}; since $\forall j \in S, i^\star \succeq_{\mathcal{P}} j$, $max(K) \in E_{max}$.